TWO-PARAMETER FAMILY OF LIQUID FLOWS ABOUT A PLATE IN THE PRESENCE OF A SMALL REVERSE FLOW

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Figure 1 shows a diagram of a two-dimensional steady-state flow of an ideal fluid about a plate. Included in the figure are the main notation and the coordinate system used here. The flow diagram differs from the normal Éfros scheme in that the critical points H_1 and H_2 behind the plate are separated along the y axis by the distance d, while the streamlines passing these points are the straight lines $y = \pm d/2$ after branching. We will refer to the band $|y| \le d/2$ as a fixed wake. Together with reverse jets of total thickness δ, this wake is realized at $x < 0$ on the second Riemann surface.

Let p_∞ and v_∞ be the pressure and velocity of the potential flow at infinity; p_0 and v_0 are the pressure in the channel and the velocity on its boundary streamline; $Q = 2(p_\infty - p_0)/\rho v_\infty^2 = (v_0/v_\infty)^2 - 1$ is the cavitation number; l is the length of the plate.

We will find the solution by using the method of Chaplygin's singularities, which was examined in adequate detail in [1] with reference to the Éfros problem. As in [1], we choose the region of the parametric variable t to be a half-circle of unit radius. Figures 1 and 2 show the coincidence of the points of the physical plane $z = x + iy$ and the plane $t = \xi + i\eta$. Here, $t(C) = ic$, $t(H_1) = t(H_2) = ih$.

Let w be the complex potential of the flow. Comparing the flow pattern being examined here and the Éfros scheme, we conclude that the expressions for dw/dt and dw/dz coincide completely in both flows:

$$\frac{dw}{dt} = Nv_0 \frac{(t^4-1)(t^2+h^2)(h^2t^2+1)}{t(t^2+c^2)^2(c^2t^2+1)^2}, \tag{1}$$

$$\frac{dw}{dz} = v_0 \frac{(t-ih)(ht-i)(t-i)}{(t+ih)(ht+i)(t+i)};$$

$$\bar{v}_0 \equiv \frac{v_0}{v_\infty} = \frac{(c+h)(1+ch)(1+c)}{(c-h)(1-ch)(1-c)}; \tag{2}$$

$$\frac{dz}{dt} = N\frac{(t+ih)^2(ht+i)^2(t+i)^2(t^2-1)}{t(t^2+c^2)^2(c^2t^2+1)^2}. \tag{3}$$

Formally, Eqs. (2) and (3) are identical to the corresponding expressions of the Éfros problem. However, in the latter the mathematical parameters h and c are connected by nonambiguity condition [1], while there is no such connection in the present case. We will express the quantities v_0, N, δ, and d through h and c.

The velocity v_0 in the reverse flows is given by Eq. (2), while the coefficient N is determined by the plate dimension l:

$$il = \int_{BOA} \frac{dz}{dt}\,dt,$$

where integration is carried out over the half-circle BOA on which $t = e^{i\theta}$ (see Fig. 2). After (3) is inserted into the integral and simple transformations are performed, we have the relation

$$\frac{l}{N} = 8\int_0^{\pi/2} \frac{\sin\theta\,(1+\sin\theta)\,(1+h^2+2h\sin\theta)^2}{(1+c^4+2c^2\cos 2\theta)^2}\,d\theta, \tag{4}$$

which is quite different from the analogous relation (23.6) in [1].

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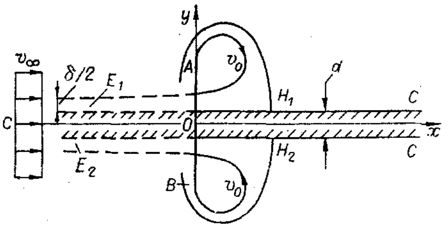

Fig. 1

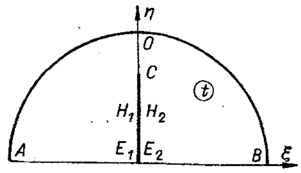

Fig. 2

The thickness of the reverse flows δ is determined by integrating (3) over an infinitesimally small one-quarter of a circle with its center at the point t = 0:

$$\bar{\delta} \equiv \frac{\delta}{l} = \pi \frac{N}{l} \frac{h^2}{c^4}. \tag{5}$$

To calculate the transverse dimension of the wake d, we integrate (3) over a circle of small radius with its center at t = ic (which corresponds to motion around a circle of fairly large radius in the physical plane):

$$id = \oint \frac{dz}{dt} dt.$$

This integral is determined by the residue of the function dz/dt at the point t = ic. Since

$$\frac{dz}{dt} = \frac{N}{(t - ic)^2} \frac{f(t)}{g(t)},$$

where $f(t) = (t + ih)^2(ht + i)^2(t + i)^2(t^2 - 1)$, $g(t) = t(t + ic)^2(c^2t^2 + 1)^2$, then the residue of dz/dt depends on the derivative of the ratio f/g at the point t = ic:

$$\frac{d}{dt}\frac{f}{g} = \frac{f}{g}\frac{d}{dt}(\ln f - \ln g).$$

Having taken the logarithmic derivative, we finally obtain

$$\bar{d} \equiv \frac{d}{l} = \pi \frac{N}{l} \frac{(c+h)^2 (1+ch)^2 (1+c)^2 (1+c^2)}{c^3 (1-c^4)^2} F(c, h). \tag{6}$$

Here

$$F(c, h) = \frac{1}{c+h} + \frac{h}{1+ch} + \frac{1}{1-c^2} - \frac{1}{c}, \tag{7}$$

while the coefficient N is given by Eq. (4). In a normal Éfros flow F(c, h) ≡ 0 (nonambiguity condition), while in the present case F(c, h), in accord with (6), determines the transverse dimension of the wake d.

Thus, the family of flows being studied is unambiguously found by assigning the parameters c and h ≤ c in accord with Eqs. (2) and (4)-(7) under the condition that the velocity of the incoming flow v_∞ and the plate dimension l are given.

Figure 3 shows theoretical dependences of the thickness of the reverse flows $\bar{\delta}$ on the transverse dimension of the fixed wake $\bar{d}$ with constant values of the velocity $\bar{v}_0$ ($\bar{v}_0$ and $\bar{d}$ are taken as independent hydrodynamic parameters). Lines 1-7 correspond to $\bar{v}_0$ = 1, 1.05, 1.1, 1.2, 1.5, 2.0, and 3.0. A characteristic feature is a monotonic and nearly linear reduction in the size of the reverse jet with an increase in the size of the wake $\bar{d}$ for all $\bar{v}_0 > 1$. The highest values of δ at v_0 = const are reached at d = 0, i.e., they correspond to the pattern of flow in accord with the Éfros scheme, the latter being a special case of the two-parameter family being ex-examined here. The minimum value δ = 0 is the same for any value of $\bar{v}_0 > 1$. It follows from (5) that this case corresponds to h ≡ 0, c ≠ 0. The coefficient N and $\bar{d}$ remain finite:

$$\frac{l}{N} = 8 \int_0^{\pi/2} \frac{\sin\theta (1 + \sin\theta)\, d\theta}{(1 + c^4 + 2c^2 \cos 2\theta)^2},$$

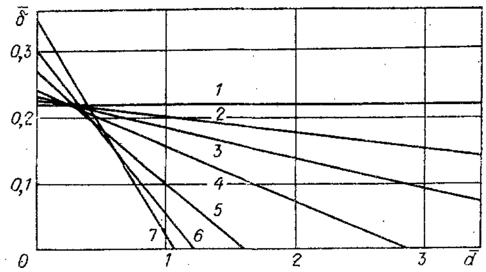

Fig. 3

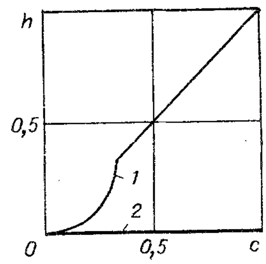

Fig. 4

$$\bar{d} = \pi \frac{N}{l} \frac{(1+c)^2(1+c^2)}{c(1-c^2)(1-c^4)^2}.$$

A simple analysis shows that at $h \equiv 0$ ($\delta = 0$), free streamlines converging from the sharp edges of the plate approach the curves $y = \pm d/2$ along tangents at the points H_1 and H_2, while the parts of the streamlines $y = \pm d/2$, $x < x_H$ disappear. These segments leave the second Riemann surface at $h \neq 0$. As a result, it turns out that the minimum value of $\delta = 0$ ($\bar{d} > 1$) is attained in the well-known Zhukov–Roshko single-parameter family of flows (see, e.g., [1]). Thus, this family is also a special case of the flows examined here.

Using the integral theorem of impulses by analogy with [1] (p. 181), we can obtain an expression for the drag coefficient if we assume that the pressure on the rear side of the plate is equal to p_0:

$$c_x = 2\bar{\delta}\bar{v}_0(1 + \bar{v}_0) + \bar{d}Q. \tag{8}$$

Here, with assigned values of Q and $\bar{d}$, the quantity $\bar{\delta}$ is a function of these two variables. As might be expected, in the limiting cases ($d \equiv 0$, $\delta \neq 0$; $\delta \equiv 0$, $\bar{d} > 1$), Eq. (8) gives the familiar expressions for the drag coefficient of a plate in a flow in accord with the Éfros and Zhukov–Roshko schemes. It was established from calculations that at $v_0 = \text{const}$, an increase in d leads to a monotonic increase in c_x, which is extremely small (it is known that the difference in the drag coefficients for flows about a plate in accord with the Éfros and Zhukov–Roshko schemes is no greater than 0.2% up to $\bar{v}_0 = 3$).

We should note certain features of asymptotic solutions in the two-parameter family in regard to Kirchhoff flow. Figure 4 shows the region of permissible values of the mathematical parameters h and c. One of the boundaries of this region (curve 1) corresponds to the family of Éfros flows in which h and c are connected by the relation $F(c, h) = 0$ ($d \equiv 0$). The transition to Kirchhoff flow ($c \to 0$) in this class is well known:

$$\bar{\delta}_* = \frac{\pi}{2(\pi+4)}, \ \bar{d}_* = 0, \ c_x = \frac{2\pi}{\pi+4},$$

just as the transition to Kirchhoff flow along boundary 2 (the family of Zhukov–Roshko flows; $h \equiv 0$, $c \to 0$):

$$\bar{\delta}_* = 0, \ \bar{d}_* \to \infty, \ c_x = 2\pi/(\pi+4),$$

i.e., it is connected with the disappearance of the reverse flow.

Now let us make the transition to Kirchhoff flow ($h \to 0$, $c \to 0$) within the class being examined.

Let $h = c^2 + \beta c^3$ ($\beta \leqslant 1$) and $c \to 0$. Then from (4) $l/N = 2(\pi + 4)$, while from (5) $\bar{\delta}_* = \pi/[2(\pi+4)]$. Simple analysis of (6) shows that $\bar{d}_* = \pi(1-\beta)/[2(\pi+4)]$, while since $Q \to 0$, then $c_x = 2\pi/(\pi+4)$.

This means that at the limit we obtain a Kirchhoff flow in which two reverse flows are separated by an amount $\bar{d}_*$ determined by the coefficient β. However, their total thickness is equal to the "Éfros" value.

Now let $h = \alpha c^2$ ($0 < \alpha \leqslant 1$) and $c \to 0$. It is easily seen that

$$\frac{l}{N} = 2(\pi+4), \ Q \approx 4(1+\alpha)c \to 0,$$

$$\bar{\delta}_* = \pi\alpha^2/[2(\pi+4)], \ \bar{d}_* = \pi/[2(\pi+4)] \quad (\alpha = 1),$$

$$\bar{d}_* \approx \pi(1-\alpha)/[2(\pi+4)c] \to \infty \quad (\alpha \neq 1).$$

However, in this case

$$c_x = \frac{4\pi\alpha^2}{2(\pi+4)} + \frac{1-\alpha}{c} \frac{4\pi(1+\alpha)c}{2(\pi+4)} = \frac{2\pi}{\pi+4}.$$

It is natural that the drag of the plate at the limit remains the same, while the magnitude of the reverse flow may be fairly arbitrary but remain within the limits $0 < \bar{\delta} \leqslant \pi/[2(\pi + 4)]$, i.e., be bounded above by the "Éfros" value. If $h \sim c^{\alpha}$ ($\alpha > 2$) and $c \to 0$, then $\delta_* = 0$, $d_* \to \infty$.

We thank G. I. Taganov for initiating this investigation.

LITERATURE CITED

1. M. I. Gurevich, Theory of Jets of an Ideal Fluid [in Russian], Nauka, Moscow (1979).

USE OF THE MODEL OF A SECOND DISSIPATIVE LAYER AND A WAKE TO DESCRIBE QUASISTEADY CAVITATIONAL FLOW ABOUT A FLAT PLATE

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The model of a second dissipative layer and a wake in [1] was used for steady flow of a viscous incompressible fluid about a flat plate within a large range of angles of attack $\alpha_{cr} < \alpha < 90°$ [2]. Comparison of the relations $c_x = f(\alpha)$ and $c_y = f(\alpha)$ obtained in [2] (see [3] also) with experimental data showed that the model underestimates c_x and c_y in this range by about 15%. This underestimation was explained in [2] by the fact that the distinctly nonsteady flow seen under experimental conditions is replaced by a quasisteady flow in the model. Moreover, the model does not consider the energy associated with pulsative motion in the near wake, which was directly confirmed experimentally in [4]. The suppression of nonsteady pulsations behind a flat plate (the experimentally-fixed reduction in the frequency of vortex shedding) in a flow at an angle $\alpha = 90°$ through the use of a splitter plate of roughly chord length located along the symmetry plate of the flow in the separation zone also leads to a reduction in c_x by about 15% at the limit, i.e., to as close an agreement between the theory and experiment as can be expected from a hydrodynamic model. If we consider that the problem of theoretically determining the drag of a flat plate located perpendicular to an incoming flow has attracted the attention of physicists and hydrodynamicists for the last two centuries, then the success of the model of a second dissipative layer and wake in regard to the solution of this problem offers hope and grounds for use of the model to solve a related hydrodynamic problem (the subject of the present investigation) – theoretical determination of the resistance force acting on a plate in a separated cavitational flow as a function of the determining parameter – the cavitation number $Q = 2(p_\infty - p_c)/\rho v_\infty^2$ (p_c is the pressure in the cavity behind the plate).

There arises the question of the need for a new (energy) approach to an old hydrodynamic problem which was theoretically described by the middle of the present century by any of four mathematical models. While differing somewhat from each other at $Q \neq 0$, at $Q \to 0$ these models approach the classical Helmholtz–Kirchhoff model $c_x = 2\pi/(\pi + 4) \simeq 0.88$ and are in fair agreement with the experimental data in the range of cavitation numbers $0 < Q < 1.0$.

Let us discuss the considerations which motivated us to develop a new approach.

First, it has long been known that separated cavitational flow is nonsteady and that it is not possible to construct a steady flow of an incompressible fluid which can reliably describe the flow observed experimentally at $Q \neq 0$ without contradicting physical reality. Since the well-known mathematical models of flow about a plate at $Q \neq 0$ are steady-state models, the drag values obtained from them can, strictly speaking, be regarded only as conditional. This conditionality is due to the effect of other bodies artificially placed in the flow on the test body in the Ryabushinskii and Zhukov–Roshko models and to the effect of the flow on other sheets of the Riemann surface in the Éfros and Tulin models.

Secondly, the well-known mathematical models ignore the existence of a fluid wake with lost momentum behind the body–cavity system. Thus, the theory loses the feedback which is present in a cavitational flow be-

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